Decomposability Investigations for Control Structure Design of Recycle Systems in the Frequency-domain

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The control structure design for process synthesis tasks becomes more difficult if recycle is present in the process to be controlled. Our previous work [1] cleared up that the control structure design for recycle systems can be decomposed into subproblems including only one unit of the investigated system. The objective of this work is to apply the decomposability in the frequency-domain. Our investigations prove that the task of the control structure design for the investigated industrial system is decomposable in the frequency domain too and this can facilitate the control structure design.

Keywords: control structure design, frequency-domain, decomposability, recycle processes

In the chemical engineering the recycling is a widely used solution to utilize material and energy more efficiently, especially in separation systems. From controllability aspects, recycle can be represented as a positive feedback in the system and it means not only process design problems, but it needs also special controllability considerations. In several cases the recycle can lead to instability and other special problems. These kind of controllability problems were exhaustively investigated [8-10]. He pointed out that the recycle loop gain basically determines the behaviour of the recycle systems and reflected to the problem called "snowball-effect" [11] in some reactor-separator systems. These results were certified by the researchers in [14], their work also reflected the importance of the control of the recycle path and the application of different controls allowed to handle the snowball-effect. Different theoretical approaches were also published. It was investigated a reactor-separator system from the point of the poles of the plant and declared the recycle as a positive feedback, too [12]. A new classification of the effects of the recycle was recommended. The correlation between the dynamic effect of the recycle and the recirculated material flows was exhaustively investigated. It was found that when the flow rate of the recycle is significantly higher than the feed flow rate, the recycle network exhibits a time-scale separation: in the fast scale the system shows fast dynamics, and in the long scale there are weak interactions. A nonlinear supervisory controller was recommended. From the point of the process design the work of Dimian et al. [15] is very considerable. They investigated the integration possibilities of process design and controllability analysis for large plants, with significant recycles and the structures with open- and closed-loops, and recommended different alternative structures and shorter recycle paths. In case of extended systems with more units and with more recycles, the control structure design becomes more difficult, and this urges exhaustive investigation for the possibility of structure decomposition. A new plantwide decomposition method which can be supported with the AHP (analytical hierarchical approach) was recomanded [16]. This method allows prioritizing the design objectives, the operability constraints, and the alternative decompositions. In this paper the process design method is based on the frequency-dependent controllability indices, and the decomposability properties of the control system are exhaustively investigated.

In our previous work [1] we successfully determined the control structure for an ethylbenzene-producing industrial system and proved the decomposability of the task of the control structure design for some 2×2 recycle systems and for the industrial system too (the system operates with two controlled and two manipulated variables). The method for the 2×2 system with hypothetical transfer function matrices are based on different load rejection simulations, and then the investigations were extended to the industrial system and the decomposability for the control structure design of the ethylbenzene-producing system was also proved. In this paper exhaustive investigations are carried out of the previously investigated ethylbenzene producing system, other different tools of control system design are applied and the investigations are extended to the frequency- and the time-domains.

Experimental part

The scheme of the investigated industrial system [6] can be seen on figure 1. The feed of the continuously stirred tank reactor are benzene (C_6H_6) and ethylene (C_2H_4), the chemical reactions can be seen in Table 1, while the flow rates are presented in Table 2. In the chemical reactor we apply a temperature of 180^0 and a pressure of 10 atm using $AlCl_3$ as catalyst. The rate constants follow the Arrhenius-law dependence, that is:

$$k_i = k_{0i} e^{-\frac{E}{RT}} \qquad (1)$$

The feed of the first column contains all of the alkylated compounds (ethylbenzene, diethyl-benzene and triethyl-benzene) and residual benzene. The first column separates the benzene which is recirculated back to the reactor. The second column separates the ethylbenzene with a high (99.9%) purity, and the third column separates the heavier components from each other. The diethyl-benzene is

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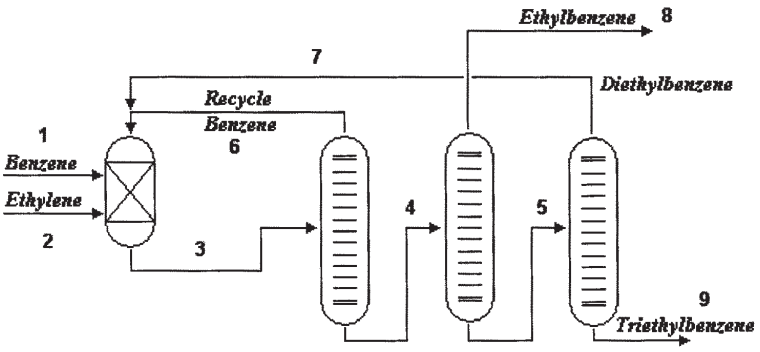


Fig. 1: The investigated system

Table 1
REACTIONS IN THE CSTR

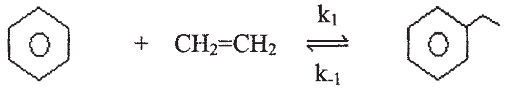
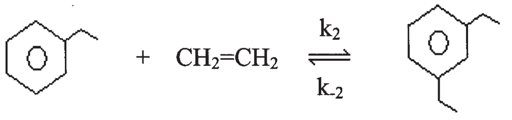
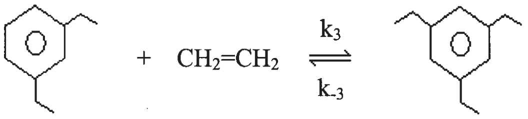
	A_1=41 m^3/kmol h A_{-1}=0.842 m^3/kmol h	$\Delta_r H_1$=-114.2 KJ/mol
	A_2=33 m^3/kmol h A_{-2}=2.12 m^3/kmol h	$\Delta_r H_2$=-104.1 KJ/mol
	A_3=17 m^3/kmol h A_{-3}=14.4 m^3/kmol h	$\Delta_r H_3$=60.2 KJ/mol

Table 2
THE MATERIAL FLOW RATES OF THE SYSTEM

Stream No.	Component(s)	Flow rate [kg/h]
1	Benzene	6600
2	Ethylene	3200
3	B, EB, DEB, TEB	50450
4	EB, DEB, TEB	13950
5	DEB, TEB	8150
6	Rec. benzene	36500
7	Rec. DEB	4150
8	EB	7350
9	TEB	2450

recirculated too, but the flow rate of this stream is significantly smaller than the flow rate of the benzene-recycle.

The conversion of the benzene in the chemical reactor is 28%, while the ethylene is totally converted, so the feed of the first column does not contain ethylene. Two recycles are applied: the distillate of the first and the distillate of the third column. The flow rates of the different streams of the system are presented in Table 2.

Effect of the recycle in the time-domain

The most significant effect of the recycle on the investigated system is the change of the time constants. The step-responses of recycle systems are slower, and on the second hand, have higher gains than without recycle. The open loop investigation is an adequate tool for preliminary investigations of the dynamics of the recycle systems and, from the results, further information can be obtained for tuning the composition control loops. During open loop simulations, composition control loops are switched off; hence the disturbances cause definitive shifts in the product compositions. Figures 2, 3 and 4 show the responses of the composition of the distillates of the three columns to feed flow rate and feed composition disturbances. Disturbances are applied at the column feeds and considering the result of some linearity-investigation, the extent of the disturbances is 1%. Bold curves show the columns' responses without recycle, while the thin curves show the responses with recycles (both of them).

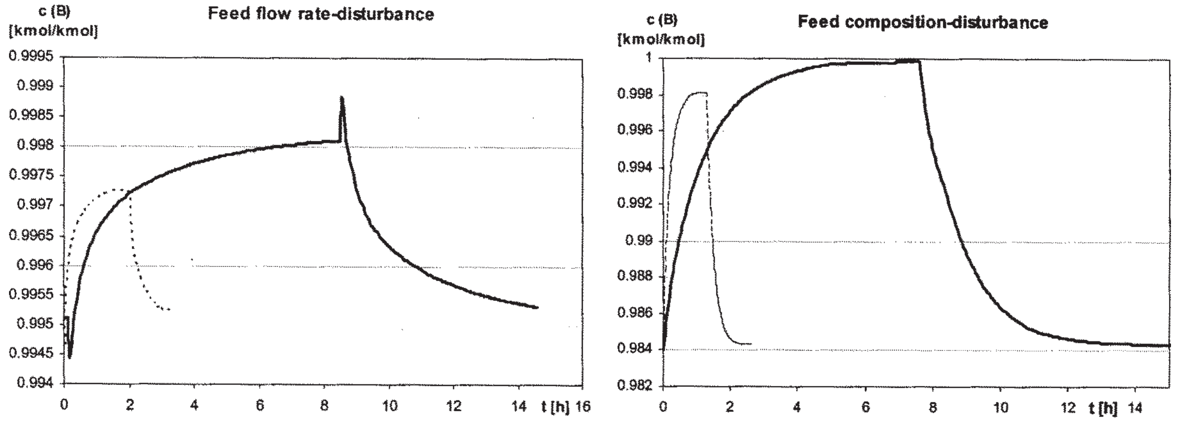


Fig. 2/a, 2/b: Step responses of the composition of the distillate of the first column-the solid line represents the system with recycle, the datshed line represents the system without recycle

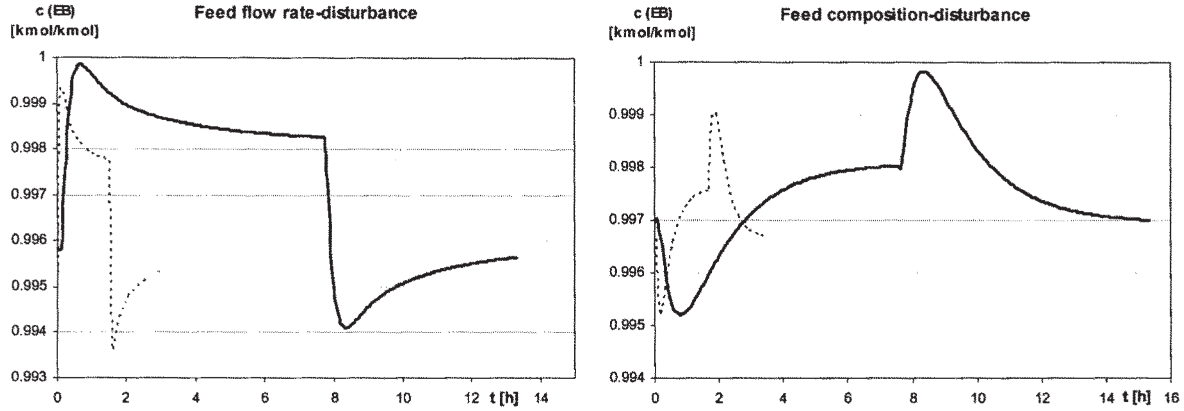


Fig. 3/a, 3/b: Step responses of the composition of the distillate of the second column - the solid line represents the system with recycle, the dashed line represents the system without recycle

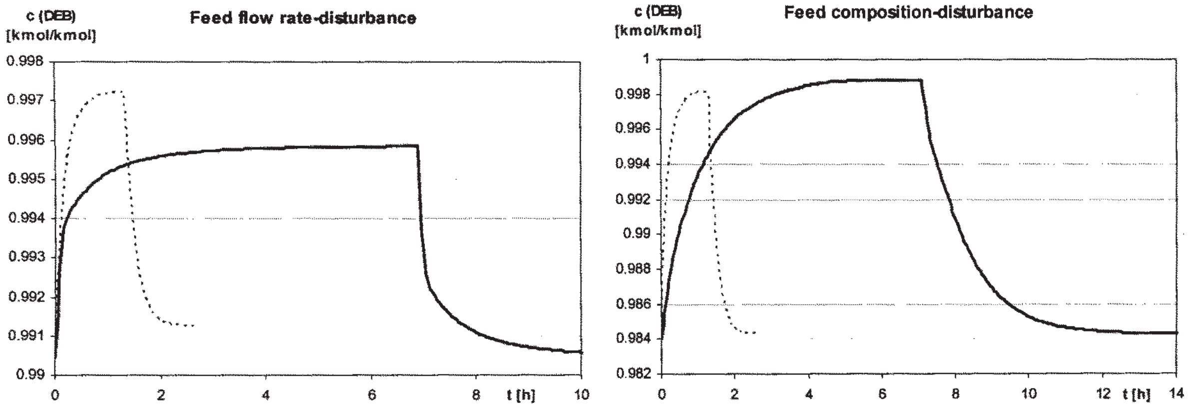


Fig. 4/a, 4/b: Step responses of the composition of the distillate of the third column the solid line represents the system with recycle, the dashed line represents the system without recycle

Compositions of the key-components are shown only. The key components are: benzene in the first, ethylbenzene in the second and diethyl-benzene in the third column.

The open-loop responses clearly show the effect of the applied recycles: in case of recycle, the time constants of the columns are 3-5 times higher than without recycle. The dynamic properties of the chemical reactor is not investigated here, only the distillation columns.

Optimal control structures

In order to keep the product-compositions at their prescribed values, composition control loops are designed for the system. The details of the design process is described in our previous work [1]: it is a load rejection based control structure design, which operates with the system properties in the time-domain. Table 3 shows the optimal control structures for each column in case of all of the possible recycles.

Table 3

THE CONTROL STRUCTURES OF THE COLUMNS (L: REFLUX RATE, R: REFLUX RATIO, B: BOTTOMS RATE, Q; REBOILER HEAT DUTY)

	1^{ST} column	2^{ND} column	3^{RD} column
No recycle	L-Q	R-Q	R-B (R-Q)
Only DEB recycle	L-Q	R-B	R-Q
Only B recycle	R-Q	R-Q	R-Q
Both of recycles	L-Q	R-B	R-Q

Decomposability analysis in the frequency-domain

The load rejection-based decomposability investigations in the time-domain [1] proved that the task of the control structure design is decomposable. Now the investigations are extended to the frequency-domain and the decomposability of the task of the control structure design is investigated based on frequency-dependent controllability indices.

In order to quantify the different composition control loops, the state space representation of the system and the frequency-dependent controllability indices are used. With the help of the Control Design Interface of Aspen Dynamics the state space matrices (A, B, C and D) are obtained and different frequency-dependent controllability indices (CN, MRI and RGA numbers) are calculated using Matlab. In this way frequency-function are obtained for the distillation columns. On Figure 5 an illustrative example can be seen: the condition number of the first column in case of both of recycles is plotted in the function of the frequency. Similar frequency-functions are determined for the investigated systems with different case of recycles.

The CN and the MRI represent the controllability and the expectable stability of the system faithfully, while the RGA numbers are closer to the steady-state representation. From the CN and from the MRI we create two modified frequency-dependent controllability indices: the average CN and the average MRI. The average CN is averaged values of all of the CN values in the whole frequency-range, while the average MRI is similarly generated by the MRI values of the investigated frequency-range. The average CN provides information the applied pairing of the investigated manipulated and controlled variables and the average MRI values show the distance of the structure from the singularity in the whole frequency-range. The unique values of these indices represent the system only at some specific frequency, but the average values give a good approximation in the whole studied frequency-range. With the help of this two modified indices representative qualitative analysis can be carried out for the investigated

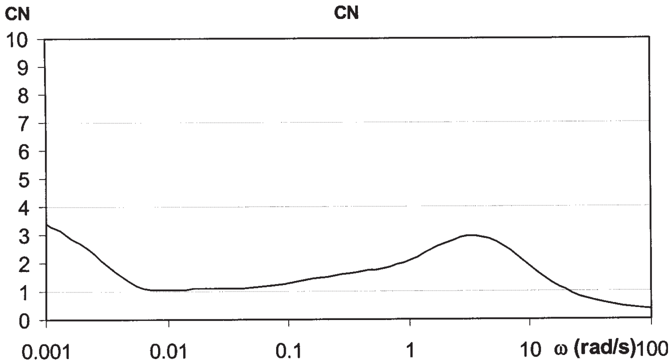


Fig. 5. a frequency-dependent controllability index (CN) in the function of the frequency – first distillation column, both of recycles (benzene and diethyl-benzene)

ethylbenzene producing recycle system in the frequency-domain. The results of these investigations are presented as a two-variable function and this methodology requires a 3D-representation. With the help of this multivariable function, the decomposability of the task of the control structure design for the global system can be certified.

Results and discussions

In order to determine the global optimal control structure, it is necessary to investigate if the individually determined optimal control structures for each column can form the overall optimum or not. If they can form the overall optimum, it is possible to determine it for the recycle system with the application of the optimal control structure of the individual units. In such a case it results that the control structure design can be decomposed. Based on load rejection investigations it is proved that the control structure design is decomposable and now, frequency-dependent controllability indices are used to perform analysis in the frequency-domain. Simultaneous investigations are carried out, considering the first and the second columns.

The first selected modified controllability index is the average MRI, which is calculated from the MRIs in the whole investigated frequency-range, it is an arithmetic average. Figure 6 shows the average MRI values for the global system. The horizontal axes contain the control structures of the first and the second columns, while the vertical axis contains the current average MRI.

On Figure 6 a monotonous surface can be seen. The optimal control structure for the whole system (which has the highest MRI) can only be constructed from the optimal control structures of the first and the second columns, every other pairing of the columns result a more unfavorable global system. The effect of the third column is not significant, because it has the smallest recirculated material flow rate (nine-times less than the recycle from the first column). Therefore the third column is represented only as a parameter here. Replacing the optimal control structure of the third column to the second best one, increases the average CN values less than 1%.

The second selected controllability index is the average CN number. This index represents the average of the measured CN values in the whole investigated frequency-range. Figure 7 shows the average CN values in case of different pairing of the control structures of the first and the second columns.

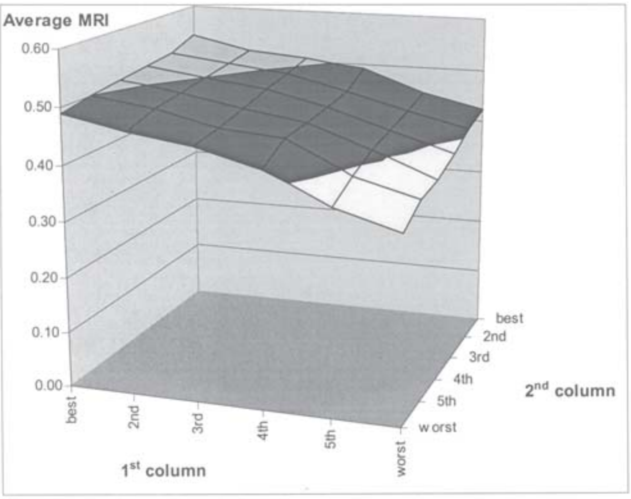


Fig. 6. The average MRI values for the global system in the case of recycles – in the function of the applied control structures of the first and the second columns

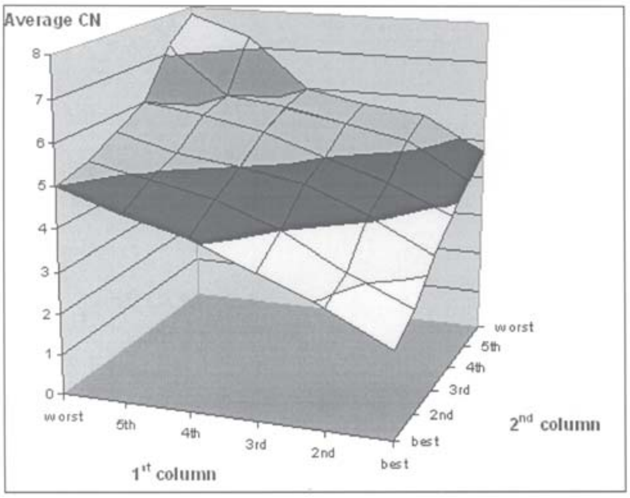


Fig. 7. The average CN values for the global system in the case of recycles – in the function of the applied control structures of the first and the second columns

The MRI values represent how far the control system is from the singularity, and the average MRI values approaches it in the whole frequency-range. More the MRI values are more the system is farer from the singularity. Each point of the MRI-surface shows the average MRI value of the global system in case of application of the first column with the control structure on the x-axis, and the second column with the control structure on the y-axis. The x and y axes show the optimality of the applied control structures. The CN numbers represent the interactions between the control loops and indicate the conditionality of the control loops. In the whole frequency range it is represented by the averaged CN here. Both of the obtained surfaces are monotonous surfaces, and represent that changing a control structure to an unfavorable one, it causes a more unfavorable behaviour for the global system concurrently. In other words, the pairing of two more unfavorable control structures never forms a more favorable global system. Similarly to the results of the MRI values, only one point can represent the optimal control structure for the global system, and it can only be formed by pairing the best control structure of the first, and best of the second columns. This fact certificates the decomposability of the task of the control structure design for this recycle system in the frequency domain, too.

The decomposability investigations in the frequency-domain reproduced the results of the load-rejection-based decomposability investigations, but in a more effective and easier way. The optimal control structures are the same, and the decomposability feature is successfully certified in the frequency- and in the time-domain too. Concerning these results we investigate the effect of the recycle on

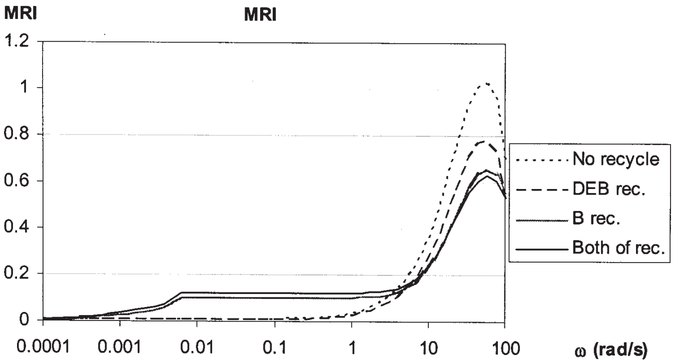


Fig. 8: The MRI values of the whole system

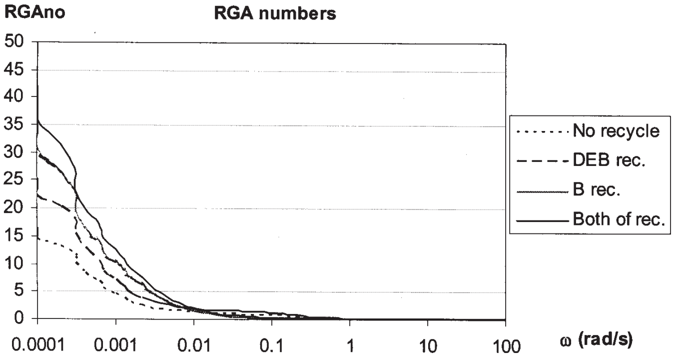


Fig. 9: The RGA numbers of the whole system

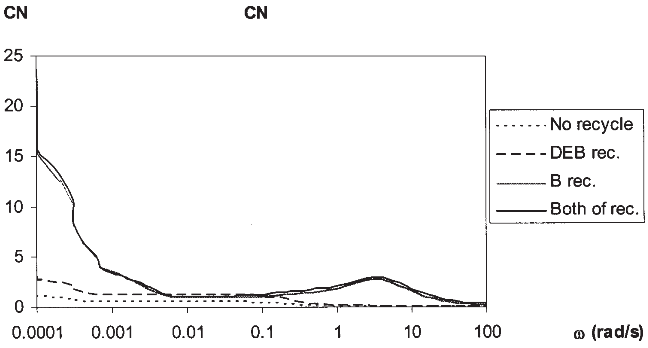


Fig. 10: The CN values of the whole system

Table 4
THE AVERAGE VALUES OF THE CONTROLLABILITY INDICES IN CASE OF DIFFERENT RECYCLES (MRI, CN AND RGA VALUS)

	MRI	CN	RGA	Stream [kg/h]
No recycle	0.86	1.45	0.44	-
DEB recycle	0.68	1.52	0.4	4150
B recycle	0.57	2	0.36	36500
Both of recycles	0.55	2.1	0.32	40650

the systems equipped with the previously determined optimal control structures.

Effect of recycle on the controllability features in the frequency-domain

The frequency-based decomposability investigations appoint the control structure for the global system. The system equipped with this control structure is investigated in the frequency-domain: MRI values, RGAno's and CN values are measured in case of different recycles. The system has two possible recycles, the benzene-recycle from the first column, and the diethyl-benzene recycle from the third column. These can also be applied independently, and the indices can be measured with both of the recycles, too. Figure 8, 9 and 10 show the different frequency-dependent controllability indices in the function of the frequency, in case of different recycles.

The correlation between the recycle flow rates and the controllability properties can be seen from the frequency-dependent indices: by increasing the flow rate of the recycled streams, the controllability indices show more unfavorable system behaviour. Table 4 shows the average values of the different controllability indices and the flow rates of the different recycled streams.

Conclusions

In this paper a frequency-domain based application is presented to design optimal control structure for an industrial, ethylbenzene-producing recycle system. The system is investigated in the frequency-domain based on different controllability indices which proves that the task of the control structure design is decomposable and proves that this is not necessary to investigate the controllability features in the time-domain, it can be completed in the frequency-domain on this easier way. The frequency dependent controllability indices clearly show the effects of the applied recycles: more the flow rates of the applied recycles are, more the frequency-dependent controllability indices are unfavorable; the systems with high flow rate of recycle streams have smaller resiliency-indices, it represents a closer state to the singularity. In the steady-state-, and the time-domain the preliminary investigations certified the correlation between the recirculated material flow rates and the values of the different controllability indices, and now these results are extended to the frequency-domain. The composition control loops are selected based on the frequency-dependent indices and the system equipped with these control loops provides the expected product compositions and is stable in each recycle cases.

Nomenclature

2×2- two manipulated and two controlled variables
B - Benzene
B - Bottom flow rate
BR - Boilup ratio
CN - Condition number
D - Distillate flow rate
DEB - Diethyl-benzene
EB - Ethylbenzene
F - Feed
G - Transfer function (or transfer function martix)
IAE - Integral Absolute Error
L - Reflux flow rate
MIMO - Multiple input, multiple output
MRI - Morari Resiliency Index
Q - Reboiler heat duty
R - Reflux ratio
Rec. - Recycle
RGA - Relative Gain Array
TEB - Triethyl-benzene
x - input signal
x_F - composition of the Feed
y - output signal

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